Duplicating Road Patterns in South African Informal Settlements Using Procedural Techniques

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Figure 1: Real world road patterns versus procedurally generated patterns.

Abstract

The formation of informal settlements in and around urban complexes has largely been ignored in the context of procedural city modeling. However, many cities in South Africa and globally can attest to the presence of such settlements. This paper analyses the phenomenon of informal settlements from a procedural modeling perspective. Aerial photography from two South African urban complexes, namely Johannesburg and Cape Town is used as a basis for the extraction of various features that distinguish different types of settlements. In particular, the road patterns which have formed within such settlements are analysed, and various procedural techniques proposed (including Voronoi diagrams, subdivision and L-systems) to replicate the identified features. A qualitative assessment of the procedural techniques is provided, and the most suitable combination of techniques identified for unstructured and structured settlements. In particular it is found that a combination of Voronoi diagrams and subdivision provides the closest match to unstructured informal settlements. A combination of L-systems, Voronoi diagrams and subdivision is found to produce the closest pattern to a structured informal settlement.

CR Categories: I.3.7 [Computer Graphics]: Three-dimensional graphics and realism; I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Geometric algorithms, languages, and systems; I.6.5 [Simulation and Modeling]: Model development—Modeling methodologies

Keywords: Procedural modeling, city modeling, informal settlement, Voronoi, subdivision, L-systems

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1 Introduction

1.1 Problem Statement

The problem addressed in this paper is the creation of feasible graphical models of road patterns found in informal settlements. In particular, the identification and use of algorithmic (procedural) techniques in the production of such models is addressed, with specific focus of identifying techniques that produce the closest matching visually apparent features to those identified in aerial photographs of informal settlements.

1.2 Background

Three-dimensional city modeling refers to the construction and representation of terrain, streets, buildings, and vegetation of urban areas. However, the complexity and size of most urban cities is sufficient to make the creation of a 3D model a challenging one. In addition, there are many social issues inherent in a modern city which affect both the appearance and evolution of areas within an urban setting.

Three-dimensional city models have applications in many areas and a broad variety of users. Applications of city modeling include planning and design [Shiode 2001] in cases where the physical topology of existing cities is modeled. However, three-dimensional city models also have applications in entertainment and virtual reality. In such cases, the true structure and layout of the city is not as important as the visual believability of the city.

Procedural modeling, as opposed to manual modeling, is the process of creating models through the use of algorithms which encode the instructions needed for the creation of a model. Manual modeling processes require direct interaction with a human modeler (using 3D modeling packages). Procedural modeling eliminates the need for direct human interaction. The user sets parameter values for the algorithms, which in many cases is the only form of interaction a user has with the system. There are a number of benefits of procedural modeling:

- Procedural modeling is parameterised, which allows for an almost infinite amount of variation of a model. Parameterisation also allows for changes to a specific model without the need to change the entire scene [Marshall et al. 1980].
- Procedural modeling promotes database amplification, which generates a large amount of data from a small set of inputs [Apodaca and Gritz 2000]. This is advantageous since large, complex models can be acquired with minimal inputs.
- Procedures have the ability to replicate processes which occur in natural systems, and lead to the creation of structures which reflect the real world.

Procedural city modeling refers to the autonomous construction of three-dimensional cities, where only a small number of initial parameters need to be provided. Such systems make use of algorithms, rather than manual specification, to define road patterns and placement of structures on a terrain. The initial parameters affect the appearance of the final outcome, and often seeded random values are used in order to generate a rich and non-uniform appearance. However, finding the correct set of parameters (and corresponding values) in order to exactly model an existing city is near-impossible, and hence procedural cities are aimed at simulations, virtual reality and entertainment applications.

One particular feature of many cities in South Africa and around the globe is that of *informal* or *spontaneous settlements*. This refers to

"dense settlements comprising communities housed in self constructed shelters under conditions of informal or traditional land tenure" [Hindson and McCarthy 1994].

Theories regarding the choice of location for such settlements have been provided, but very little is said about the internal road structure and layout of these settlements. This paper analyses the phenomenon of informal settlements using two major South African urban complexes as examples. Aerial photographs of Johannesburg (and surrounding municipalities) and Cape Town are used for the analysis. In particular, the road patterns and layout of shacks which have formed within such settlements are analysed, and various procedural techniques presented that model the identified phenomena.

1.3 Overview

This paper is structured as follows. Section 2.1 presents a review of work done in procedural city modeling. Section 2.2 presents observations made in literature regarding the formation and structure of informal settlements. Section 3.1 analyses aerial photographs of informal settlements, and motivates schemes to be used for modeling settlement patterns. Implementation details, as well as a discussion regarding the success of these schemes are presented in Section 3.2 and Section 3.3 respectively. Finally a summary of findings is presented in Section 3.4.

2 Related Work

2.1 Procedural City Modeling

Procedural city modeling techniques make use of algorithms and rules to generate street patterns, and position buildings. The key to this approach is a minimal set of input parameters. A number of tools exist that facilitate the generation of convincing city road patterns:

- Input Maps: factors such as population density and elevation are represented as two dimensional input maps, and passed as input into a procedural system [Parish and Muller 2001; Sun et al. 2002]. The values derived from these maps serve as parameters for the procedural techniques over the terrain.
- **Templates:** Templates use information provided by input maps to parameterise a set of patterns commonly found in cities, such as Voronoi diagrams, raster patterns and radial patterns [Sun et al. 2002].
- **L-Systems:** L-systems are used to generate road patterns based on local and global patterns specified in the input maps [Parish and Muller 2001]. Once road patterns have formed, the areas between the roads are divided into *lots* upon which building models are placed.
- Non-oriented graphs: graphs are used to represent street patterns, where the vertices of the graph represent crossroads, and edges represent streets. Edges are removed and vertices displaced to introduce variation [Marvie et al. 2003]. Buildings are created by extruding the regions between the edges [Greuter et al. 2003].

2.2 Sociological Studies of Informal Settlements

This section briefly presents evidence from investigations regarding the structure and layout of informal settlements in South Africa. In particular features such as settlement location and structure, as well as housing characteristics are discussed.

A number of assertions are made by Dwyer [1975] regarding the location of informal settlements in urban areas. One trend identified is that informal settlements typically develop on any empty land, governmental or otherwise, in and around urban complexes. This trend can be easily motivated by examples in the Ekurhuleni municipality, to the East of Johannesburg, where settlements have rapidly formed on mining land which recently became vacant due to the removal of mine-dumps. This trend is motivated primarily by the fact that few dwellers in informal settlements can afford the costs of commuting, and hence vacant sites close to industrial areas are especially valued. Additionally, it is often claimed that migrants first settle in centrally located settlements to be close to areas of employment, but relocate to more spacious areas on the peripherals of the city when more financial stability is achieved [Drakakis-Smith 1987].

The structure of informal settlements in urban areas is more difficult to define. Dwyer [1975] indicates that often regular grid formations occur in informal settlements, but equally often the building patterns are too chaotic to be rationalised. Manona et al. [1996] distinguishes between two types of settlement patterns based on studies of informal settlements in the Eastern Cape. Poor quality structures, and chaotic patterns tend to occur when people erect structures knowing that they will leave and build elsewhere. However, when people have clear intentions of settling, more formal patterns and higher quality structures are erected. In these cases, plots must be requested from a local committee - resulting in neat arrangements with accessible streets. Drakakis-Smith [1987] highlights an important feature which influences many South African informal settlements - aided self-help programmes. These schemes aim to upgrade informal settlements by providing basic essential infrastructure such as water, sewerage and electricity which are provided on prepared lots. This results in the formation of regular street patterns.

Housing structures in squatter settlements are typically built from ad hoc materials ranging from wood, to corrugated iron. However, according to Hindson and McCarthy [1994] and Manona et al. [1996] this is not always the case. In fact, often the condition of housing in informal settlements tends to improve over time, resulting, in some cases, in mud-brick structures with corrugated iron roofs. However, temporary structures tend to be of low quality, containing only a single room, and are in the majority according to Manona et al. [1996], where a third of the households studied contain only one room. In addition, the aerial photographs of informal settlements around Johannesburg and Cape Town reveal that the general structural shape of shacks is predominantly cube-shaped (see Figures 2, 3 and 4).

3 Identification and Evaluation of Modeling Techniques

This experimentation has a number of goals:

- Identify a subset of visual patterns found in informal settlements from South Africa
- 2. Suggest and motivate the use of procedural techniques which may be able to emulate such patterns
- 3. Implement the identified techniques
- Provide a qualitative evaluation of the believability of each scheme in comparison with actual photographs of informal settlements

The first two goals are discussed in Section 3.1. Issues relating to the implementation of the identified techniques are discussed in Section 3.2, with evaluations presented in Section 3.3.

3.1 Algorithm Analysis

A number of aerial photographs of informal settlements from the Johannesburg region and Cape Town have been analysed. Figures 2, 3 and 4 are aerial photographs of informal settlements near Johannesburg and Cape Town [Google Maps Beta 2005]. Figures 2 and 3 represent unstructured settlement where no external planning or improvement efforts have been made. Figure 4 however, represents a structured settlement which formed after the provision of basic infrastructure such as sewerage and electricity.

3.1.1 Features

The most striking visual feature inherent in the figures are the wide pathways between clusters of shacks. These pathways are henceforth referred to as major-road patterns. The widest major-road patterns are manually identified and highlighted in white for clarity. Since we know nothing more about the dynamics of the settlement than what is visually apparent in the image, the criterion used to distinguish these patterns is merely the exaggerated perceived breadth of the major-roads. As noted in Section 2.2 a highly irregular pattern is present in both unstructured settlements (Figures 2 and 3), however where basic services are provided a very structured street pattern forms (Figure 4). More difficult to identify is what we term minor-road patterns, which represent auxiliary pathways between the major pathways. The density of shacks between major-roads is also a feature which distinguishes structured and unstructured settlements, with unstructured settlements being more densely populated. Crossroads tend to be more radial in nature in unstructured



Figure 2: Unstructured informal settlement near Johannesburg



Figure 3: Unstructured informal settlement near Cape Town.

settlements, while the more structured settlements exhibit perpendicular intersections. Finally, parallel roads are not visible in the unstructured settlements. Table 1 presents a summary of the perceived visual features of informal settlements based on the three aerial photographs.

The following procedural tools have been identified which have the potential of capturing the identified features of informal settlements:

- Voronoi Diagrams: The patterns formed by major pathways within the informal settlements in Figures 2 and 3 are highly suggestive of the regions formed by Voronoi diagrams. Given a set of reference seed-points on a plane, a Voronoi diagram partitions the plane into a number of regions, where each region is the locus of points which are closest to each reference point [Preparata and Shamos 1985]. The result is a number of irregular polygons (see Figure 5 for an example). If the edges of these polygons are interpreted as streets, the results are similar to the irregular patterns in Figures 2 and 3.
- **Subdivision:** The seeming regularity of minor-road patterns of the informal settlements in the aerial photographs suggests the use of subdivision as a modeling tool. Subdivision is the



Figure 4: Structured settlement near Johannesburg, where basic services such as sewerage and electricity are provided.

Features	Unstructured	Structured	
Major-road pattern	Irregular	Regular	
Minor-road pattern	Semi-regular	Semi-regular	
Density of shacks	High	Moderate	
Crossroads	Radial	Perpendicular	
Road orientation	Star	Parallel	

Table 1: Perceived visual features of informal settlements

process of increasing the resolution of a mesh by adding new vertices. The locations of these new vertices are calculated in terms of the existing mesh vertices at each stage of subdivision. An example of subdivision is presented in Figure 6, which is a plane with four levels subdivision.

- Noise: To introduce some randomness into the layout of the informal settlements, Perlin noise [Perlin 2002] is used to distort the regular pattern created by the subdivision process. Figure 6 uses noise to displace the vertices, giving an irregular appearance.
- **L-Systems:** As an alternative to subdivision L-systems are proposed as a scheme to model the seeming regularity of minor-road patterns. An L-system is specified by a *grammar*, to which *productions* are applied a variable number of times, resulting in a dense tree-like structure. If regular angles are chosen at branch-junctions then a very regular pattern forms (see Figure 7).

The individual use of each of the above techniques is not sufficient to model convincing informal settlements. Thus combinations of the above tools are created based on observations made from the aerial photographs. Voronoi diagrams are chosen to represent the major street structure, with the initial choice of seed points differentiating between structured and unstructured patterns. The more random the choice of these points, the more irregular the resulting polygons. Within each Voronoi region, either subdivision or L-systems are used to produce grid-like patterns. Regular grids are avoided by displacing vertices using noise. Table 2 presents a summary of the experiments conducted.

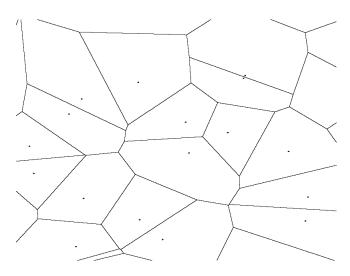


Figure 5: Voronoi Polygons

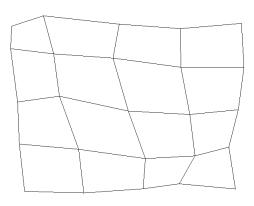
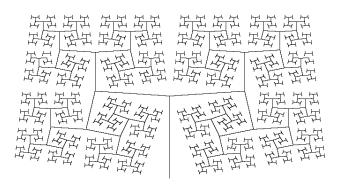


Figure 6: Subdivision with noise

	Voronoi Seed-points	Minor Road Pattern
A	Random	Subdivision (noise)
В	Random	L-System
		(randomised angles)
С	Random	Subdivision (noise-free)
D	Random	L-System
		(consistent angles)
Е	Regular Grid	Subdivision (noise)
F	L-System	Subdivision (noise)
	(consistent angles)	

Table 2: Combinations of procedural tools



$$R = 1.456$$

 $\omega = F(2)$
 $p_1 = F(s) \to F(s)[+A(s/R)][-A(s/R)]$

where F(x) represents a line segment of length x in the current direction. The initial value of s is 2. + and - refer to positive and negative rotation about the y-axis respectively.

Figure 7: L-system grammar

3.2 Procedural Model

The procedural tools used for these experiments are implemented in the following manner:

- **Voronoi Diagrams:** An initial set of points (regular or random) are generated on the x-z plane as reference seed points for the Voronoi diagrams. The corresponding Voronoi diagram is generated using the QVORONOI application of the QHull package [Barber et al. 1995]. Any resulting polygon containing a point-at-infinity is removed, and points that fall beyond a specified boundary are shifted to fall within the bounds.
- Subdivision: √2-subdivision, presented in [Li et al. 2004], is implemented for these experiments. The mesh to be subdivided can be of arbitrary topology and consist of either triangles or quadrilaterals. The scheme has been altered to maintain the area of the mesh passed to it, by not displacing the original vertices. The density of subdivision is maintained between regions by subdividing regions a number of times proportional to their respective areas. Another modification to the subdivision scheme is the addition of noise to displace the x− and z−coordinates of each newly created vertex.
- L-Systems: The class of L-system implemented is as described by Prusinkiewicz and Lindenmayer [1990], with the grammar presented in Figure 7. A regular pattern is produced by setting the angle of rotation to 85 degrees. An irregular pattern is produced by randomising this angle within a restricted range.

The generation of procedural informal settlements is achieved as follows. The initial set of reference points for the Voronoi diagram is generated (random or regular), and the corresponding set of Voronoi polygons is produced. Where subdivision is used for minor-road patterns, each polygon is treated as an individual mesh which is subdivided a number of times (relative to the area of the polygon). Each resulting vertex in the mesh is used as a placement location for a shack in the settlement. Where L-systems are used as minor-road patterns, the structure is grown within the polygon by

applying production rules a number of times. Any vertices falling outside the polygon are discarded. The endpoints of the L-system are used as placement locations for the shacks. L-systems are also used to generate major-road patterns by producing a regular set of reference points for the Voronoi diagram.

In order to achieve major-road patterns (in the form of gaps between shacks) in the settlement scheme, any shack which falls too close to the edges of the Voronoi polygons are discarded.

The shape and texturing of shacks is of minor importance in these experiments. Since shacks are predominantly cube-shaped, each shack in the model is represented as an untextured cube. Images are rendered using OpenGL.

3.3 Evaluation

3.3.1 Experiment A

Figure 8 presents an unstructured informal settlement using 80 random Voronoi seed-points, with each Voronoi region adaptively subdivided according to the area of the region. Each shack location is displaced according to a noise function. Additionally the orientations of the shacks are adjusted so that at least one face of the shack is aligned with the closest Voronoi boundary.

A highly irregular major-road pattern emerges from this model, with unpredictable crossroads and junctions. The major-road pattern is clearly visible and is greatly enhanced by the alignment of the shack faces with the closest road. Minor-road patterns are more difficult to identify. The distribution of the shacks is uniformly dense. There is no pattern to the structure of the roads, with parallel roads scarcely occurring. It is possible however to identify instances of radial road patterns. This image is juxtaposed with an aerial photograph of an informal settlement found in Cape Town. Features such as radial intersections, and the lack of parallel lines, is present in both images.

3.3.2 Experiment B

Figure 9 presents a combination of Voronoi regions (from 80 random Voronoi seed-points) and L-systems within these regions. In some areas there are emerging major-road patterns, but generally the spaces between clusters of shacks are unrealistically large. This can be explained by the nature of the grammar used for the L-system, which tends to leave large gaps between child branches. Barring the large gaps between regions, the density of shacks is largely uniform. No minor-road patterns can be identified within the regions, and the shack orientations are highly irregular. A similar pattern is discernible in the accompanying aerial photograph of an informal settlement near Kempton Park, East of Johannesburg. This area is located on the urban fringe, and contains a number of smallholdings, marshes, and pans. The large gaps between clusters of shacks have been darkened in the photograph, and produce similar patterns to those found in the generated image.

3.3.3 Experiment C

Figure 10 presents a more structured settlement than the previous two experiments. A Voronoi diagram is used with a smaller number of regions than in Experiment A and B (40 seed-points) and shack locations are not displaced during subdivision. However, the orientation of shacks is still modified to face the nearest roads. Once again major-road patterns emerge very clearly, but in this case

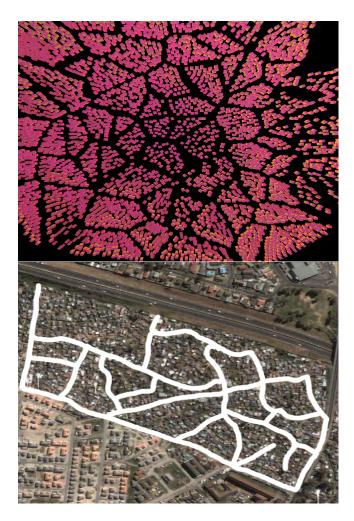


Figure 8: Experiment A: Random Voronoi seed, Voronoi regions subdivided with noise

minor-road patterns are more distinguishable than in previous experiments. The shack placement is very dense, and very uniform, giving the impression of structure. The criticism of this model is that there are regions of shacks which are too regularly distributed, displaying placement patterns which cannot be seen in the aerial photographs of informal settlements provided. Although not as irregular as the procedural approach, the real world example (settlement in Cape Town) in the figure does exhibit similar density.

3 3 4 Experiment D

Figure 11 presents another attempt at a more structured settlement. A randomised Voronoi diagram (with 40 random Voronoi seedpoints) is used, and a non-randomised L-system (branching angles of 90 degrees) is used within the Voronoi regions. The problem with this model is the sparseness of shack placement. This is a result of the L-system grammar, which grows too rapidly beyond the bounds of the Voronoi regions. The sparseness of shacks is not conducive to the formation of clear major-road patterns. The distribution of shacks within the regions is largely regular, often forming a clear grid pattern. As shown in the photograph, similar patterns have been discovered in an informal settlement in Cape Town.

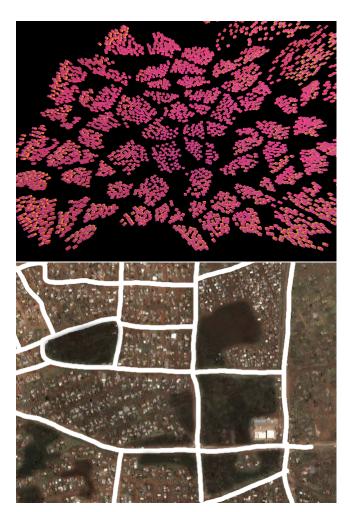


Figure 9: **Experiment B:** Random Voronoi seed, Voronoi regions contain L-systems (random angles)

3.3.5 Experiment E

Figure 12 presents the results of a Voronoi scheme based on regularly distributed seed-points. This results in a major-road structure which has a typical grid pattern. Irregularity is still maintained by employing noise in the subdivision of the Voronoi regions. Minor-road patterns do not emerge clearly. The density of shacks is not uniform, even though the size of the Voronoi regions are uniform. This is attributed to the noise applied to the vertices during the subdivision process. The criticism of this model is the overarching uniformity of the major-road patterns. However, such patterns do occur, as indicated by the corresponding aerial photograph of a settlement in Cape Town.

3.3.6 Experiment F

Finally, Figure 13 presents a hybrid of procedural methods. The regular L-system is used to generate the seed points for the Voronoi diagram, and the resulting Voronoi regions are subdivided with noise displacements. This figure provides a good example of a structured informal settlement. Major-road patterns such as parallel roads and perpendicular cross-roads are clearly visible. However, the major-road pattern is not totally regular, and there exist a number of diagonal roads, and radial intersections. The density



Figure 10: **Experiment C:** Random Voronoi seed, Voronoi regions subdivided without noise

of the shacks is also uniform in all the regions, and their orientations are convincingly regular. The main contributor of irregularity to this model is the use of noise, which has sufficiently perturbed the placement of shacks to reduce the overall impression of structure. The only criticism of the results produced by this model is the perceived symmetry, which is an artefact of the L-system which was used to create the Voronoi seed-points. Symmetry aside, this model produces many of the parallel roads and radial intersections which are noticeable in the corresponding aerial photograph of a settlement near Johannesburg.

3.4 Observations

Table 3 presents a summary of the qualitative comparisons made regarding the features identified in each experiment.

The conclusions according to Table 3 are as follows:

- Experiment A produces the most convincing unstructured pattern:
- Experiment E produces the most convincing structured pattern:
- Experiment F produces a combination of structured and un-

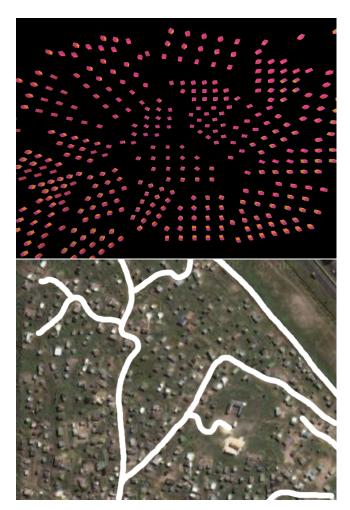


Figure 11: **Experiment D:** Random Voronoi seed, Voronoi regions contain L-systems (regular angles)

structured patterns, including parallel roads and radial intersections;

- Voronoi diagrams produce convincing major-road patterns in all cases
- Minor-road patterns are not readily identifiable when subdivision (with noise) is used. This is attributed to the large amount of variability caused by noise, the density of shacks, as well as variation in orientations;
- The formation of perpendicular and radial crossroads is successfully achieved using combinations of Voronoi diagrams and L-systems;
- Generally, subdivision works better than the chosen L-system for populating the Voronoi regions. This could be remedied choosing a different grammar and set of parameters for the L-system.

It is important to note that the conclusions presented here regarding the closest matching settlement pattern are qualitative in nature, and subjective to the viewer of the images. Further work must be undertaken to identify quantitative measurements which can be used to differentiate between structured and unstructured settlements. Furthermore, a larger base of aerial photography will be beneficial in classifying various types of informal settlements.

	Major-roads	Minor-roads	Density	Crossroads	Road Orientation
Aerial (Unstructured)	Irregular	Semi-regular	High	Radial	Star
Aerial (Structured)	Regular	Semi-regular	Moderate	Perpendicular	Parallel
Experiment A	Irregular	Unidentifiable	Uniformly high	Radial	Star
Experiment B	Irregular (gaps)	Unidentifiable	High (gaps)	Radial	Star
Experiment C	Irregular	Very structured	Uniformly High	Radial	Star
Experiment D	Unidentifiable	Grid	Low	Unidentifiable	Unidentifiable
Experiment E	Grid	Irregular	Moderate	Perpendicular	Parallel
Experiment F	Regular	Irregular	Moderate	Combination	Combination

Table 3: Summary of findings



Figure 12: **Experiment E:** Regular Voronoi seeds, Voronoi regions subdivided with noise

4 Conclusion

The creation of feasible graphical models of road patterns is possible with the use of procedural techniques. Voronoi diagrams, subdivision, and L-systems are identified as procedural tools that result in such patterns, and some success has been achieved in simulating patterns found in actual informal settlements.

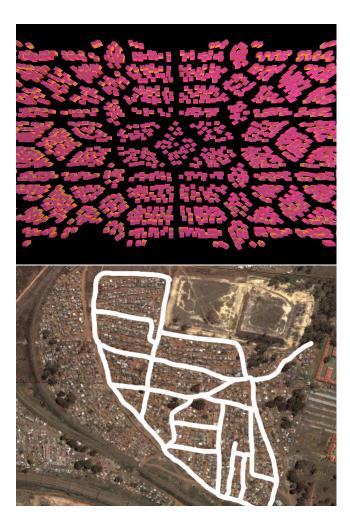


Figure 13: **Experiment F:** Regular L-systems used to generate Voronoi seeds, Voronoi regions subdivided with noise

4.1 Contributions

Two novel contributions are made. Firstly, the inclusion of informal settlements into the context of city modeling was previously an unexplored area. Secondly, novel combinations of procedural techniques (including Voronoi diagrams, subdivision, and L-systems) for city modeling are demonstrated.

4.2 Further Work

Further work includes adding a time dimension to the procedural techniques in order to simulate the growth of such settlements. Additionally, other combinations of procedural techniques can be tested (for example fractals), with the inclusion of global parameters such as water availability and land topography. A more detailed and theoretically grounded set of features should be extracted by analysing a larger set of aerial photography and settlement literature.

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